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Quantum theory of frequency shifts of an electromagnetic wave interacting with a plasma

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In this paper we calculate the frequency shift induced on a photon by the interaction with a low density electronic plasma. The technique is the standard perturbation theory of quantum electrodynamics, taking into account the many body character of the plasma. The shift in the nonrelativistic approximation is shown to be blue. Besides the quantum shift, the known classical effects and the correct temperature dependence are also obtained. Finally the limits of the approximations used are discussed. [S1063-651X(97)07305-4]

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I. INTRODUCTION

The propagation of electromagnetic waves through plasmas has been extensively studied from a classical viewpoint. This is usually justified in most cases by the physical conditions regarding the plasma temperature and density, at least when the considered wavelengths are not too short.

In the classical approach the plasma operates as a sort of active filter, both absorbing and distorting electromagnetic waves in various ways depending on the plasma parameters, its homogeneity, the presence of an external magnetic field, and of course the frequency of the wave. To obtain an idea of this complicated methodology and of the kind of results one obtains it is useful to have a look at Ref. [1]. It is remarkable that, according to the different conditions, various frequency shifts arise.

Another classical approach, particularly fit for optical, or shorter, wavelengths, is that which considers the interaction of the electromagnetic wave with a fluctuating medium as a scattering by inhomogeneities inside it. This analysis has been made in Refs. [2–4] and leads again to frequency shifts of the incoming radiation depending now also on the scattering angle. In particular, a blueshift is found for forward scattering, as a manifestation of the Rayleigh scattering.

In principle, however, the quantum aspects of the interaction of the wave with the plasma should not be overlooked. This is likely to be especially true in some situations of astrophysical or even cosmological interest, where the density of the plasma is rather low, and the distances are so high as to allow small effects to pile up.

In this paper we shall precisely investigate the quantum effects using standard quantum electrodynamics and a perturbative treatment, as outlined in Sec. II. The method will also produce the known classical features of the propagation of electromagnetic waves through a plasma, as we show in Sec. IV. On the quantum side, in Sec. III we take into account the many body nature of a plasma that entrains the appropriate fermionic statistic. In the nonrelativistic limit we find, for a low density locally homogeneous (i.e., homogeneous on the scale of a few wavelengths) plasma, a blueshift of the photon frequency (this has the same sign as that of the classical results in [2–4], though now the effect has a completely different origin). Finally in Sec. V, we discuss the validity conditions of our approximations, in particular those concerning the possibility of overlooking relativistic corrections. Just to fix ideas and exemplify our low density plasma, we refer, in various parts of the paper, to numerical values of the parameters of the order of those valid for the solar corona, i.e., a number density $\sim 10^6$ electrons/cm^3 and a temperature $\sim 10^6$ K.

II. BASIC ASSUMPTIONS AND OUTLINE OF THE METHOD

We consider a situation where an electromagnetic wave, plane and monochromatic, coexists with a plasma of electrons with a numerical density distribution $n(\mathbf{x})$. The propagation is along the x axis.

The ''unperturbed'' state is obtained when the coupling between the wave and the plasma is set to zero; consequently, the energy distribution of the electrons is that of a fermionic plasma of temperature T and density n restrained into a potential well, and the wave has a frequency ω.

Let us now switch on the coupling and, assuming the interaction energy to be small, determine the shift in the energies as a perturbation of the ''initial'' (i.e., uncoupled) situation. Supposing that the interaction is set up in a finite time lapse and comparing the situations at $-\infty$ in time with that at $+\infty$, we mimic the actual process of an incoming plane wave of frequency ω and an outgoing one of frequency $\omega+\delta\omega$.

The technique actually used to compute the shift in the frequency of the wave is that of the time independent perturbations. The interaction Hamiltonian is

$$H_I=-\frac{e}{c}\int \mathbf{j}\cdot\mathbf{A}\,d\mathbf{r}, \tag{1}$$

where $\mathbf{A}$ is the vector potential operator of the wave and $\mathbf{j}$ is the current density operator appropriate for this problem; the volume integral is limited to the confinement region of the plasma. The complete nonrelativistic expression for $\mathbf{j}$ when an electromagnetic interaction is present is [5]

$$\mathbf{j}=-\frac{i\hbar}{2m}(\Phi^\dagger\boldsymbol{\nabla}\Phi-\Phi\boldsymbol{\nabla}\Phi^\dagger)-\frac{e}{mc}\Phi^\dagger\Phi\mathbf{A}. \tag{2}$$

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Cast in the second quantization formalism, the interaction Hamiltonian is [6]

$$H_I = H_I' + H_I'', \tag{3}$$

$$H_I' := \left(\sum_{\mathbf{k}} b_{\mathbf{k}} \otimes \sum_{\mathbf{q},\mathbf{q}'} g^{\mathbf{k}}_{\mathbf{q},\mathbf{q}'} a^\dagger_{\mathbf{q}} a_{\mathbf{q}'} + \text{H.c.} \right), \tag{4}$$

$$H_I'' := \frac{e^2}{2mc^2} \frac{\hbar}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}'} \frac{\delta_{\mathbf{k}-\mathbf{k}',\mathbf{q}-\mathbf{q}'}}{\sqrt{\omega\omega'}} b^\dagger_{\mathbf{k}'} b_{\mathbf{k}} \otimes a^\dagger_{\mathbf{q}'} a_{\mathbf{q}}, \tag{5}$$

where we defined

$$g^{\mathbf{k}}_{\mathbf{q},\mathbf{q}'} = \frac{e}{m} \left(\frac{\hbar^3}{2V\omega} \right)^{1/2} \mathbf{q}' \cdot \mathbf{u}_{\mathbf{k}} \delta_{\mathbf{k},\mathbf{q}'-\mathbf{q}}. \tag{6}$$

The $b^\dagger$'s and the b's are the bosonic creation and annihilation operators associated with the photons; the a's are fermionic (electronic) operators. The energy of the photon is $\hbar\omega$, its momentum is $\hbar\mathbf{k}$, and of course $\omega = ck$; the polarization of the photon is expressed by the unitary vector $\mathbf{u}_{\mathbf{k}}$; $\mathbf{k}$'s and $\mathbf{q}$'s, respectively, are photonic and electronic wave vectors; primes denote intermediate state variables; finally, the unessential spin and polarization indices have been dropped, though an average over them in the final formulas is performed.

The last term in Eq. (2) is usually omitted under the explicit, and sometimes implicit, assumption it makes contributions small with respect to those coming from the first. The frequency shift of the external photon can be written in the form

$$\delta\omega = \delta\omega' + \delta\omega'',$$

where $\delta\omega'$ and $\delta\omega''$, respectively, are the contributions due to H_I' and H_I''. In this paper we calculate $\delta\omega$ for a low density plasma, with a perturbative treatment up to order $\alpha = e^2/\hbar c$ (this is equivalent to keep the first order term in H_I'' and the second in H_I').

III. QUANTUM FREQUENCY SHIFT

We first consider the contributions to the shift due to H_I'. Let H_{em} and H_P be the electromagnetic field and the plasma free Hamiltonians; $\mathcal{H}_{\text{em}}$ and $\mathcal{H}_P$, are the bosonic Fock space associated with the photons and the fermionic Fock space associated to the electrons, respectively; $\Phi_{\mathbf{k}} \in \mathcal{H}_{\text{em}}$ and $\Psi_{\mathbf{q}} \in \mathcal{H}_P$ are the one particle electronic and photonic wave functions.

We calculate the shift $\Delta E_{\omega,\mathbf{q}}$ of the value of the energy of the state $\Phi_{\mathbf{k}} \otimes \Psi_{\mathbf{q}}$ of $\mathcal{H}_{\text{em}} \otimes \mathcal{H}_P$. Since $\Phi_{\mathbf{k}} \otimes \Psi_{\mathbf{q}}$ is an eigenvector of $H_{\text{em}} + H_P$, we can use the second order perturbation theory (the first order energy shift is zero). The energy shift of the state i is

$$\Delta E_i = \sum_{j \neq i} \frac{(H_I)_{ij} (H_I)_{ji}}{E_i - E_j} (1 - \nu_j), \tag{7}$$

where j labels any (normalized) eigenvector of $H_{\text{em}} + H_P$, and ν_j is the probability that the state j is occupied by an electron.

The only nonvanishing contributions are those with respect to j states of the form $j_1 := \Phi_{\mathbf{0}} \otimes \Psi_{\mathbf{q}'}$,

$$j_2 := \left(\frac{1}{\sqrt{2}} \Phi_{\mathbf{k}} \otimes \Phi_{\mathbf{k}'} \right) \otimes \Psi_{\mathbf{q}'},$$

or

$$j_3 := \left(\frac{1}{\sqrt{2}} \Phi_{\mathbf{k}'} \otimes \Phi_{\mathbf{k}} \right) \otimes \Psi_{\mathbf{q}'}$$

($\Phi_{\mathbf{0}}$ is the electromagnetic vacuum). It is easy to show that

$$(H_I')_{j_1 i} = g^{\mathbf{k}}_{\mathbf{q},\mathbf{q}'},$$

$$(H_I')_{j_2 i} = (H_I')_{j_3 i} = \frac{1}{\sqrt{2}} g^{\mathbf{k}'}_{\mathbf{q}',\mathbf{q}}.$$

So the energy shift induced by H_I' on $\Phi_{\mathbf{k}} \otimes \Psi_{\mathbf{q}}$ is [7]

$$\Delta E'_{\omega,\mathbf{q}} = \Delta E^{(1)}_{\omega,\mathbf{q}} + \Delta E^{(2)}_{\omega,\mathbf{q}}, \tag{8}$$

where

$$\Delta E^{(1)}_{\omega,\mathbf{q}} := \sum_{\mathbf{q}'} (1 - \nu_{q'}) \left[\frac{|g^{\mathbf{k}}_{\mathbf{q},\mathbf{q}'}|^2}{\hbar\omega + \epsilon_q - \epsilon_{q'}} - \frac{|g^{\mathbf{k}}_{\mathbf{q}',\mathbf{q}}|^2}{\hbar\omega + \epsilon_{q'} - \epsilon_q} \right], \tag{9}$$

$$\Delta E^{(2)}_{\omega,\mathbf{q}} := \sum_{\mathbf{k}'} \sum_{\mathbf{q}'} (1 - \nu_{q'}) \frac{|g^{\mathbf{k}'}_{\mathbf{q}',\mathbf{q}}|^2}{\epsilon_q - \epsilon_{q'} - \hbar\omega'}, \tag{10}$$

where, for a Fermi gas, in the low density approximation,

$$\nu_q = \frac{1}{\frac{1}{z} e^{\beta\epsilon_q} + 1} \simeq z e^{-\beta\epsilon_q}. \tag{11}$$

In order to obtain the energy shift, we have to take the mean value of $\Delta E'_{\omega,\mathbf{q}}$ with respect to the possible states of the electrons $\Psi_{\mathbf{q}}$. We have

$$\Delta E'_{\omega} = \Delta E^{(1)}_{\omega} + \Delta E^{(2)}_{\omega},$$
$$\Delta E^{(1)}_{\omega} := \sum_{\mathbf{q}} \nu_q \Delta E^{(1)}_{\omega,\mathbf{q}}, \tag{12}$$

$$\Delta E^{(2)}_{\omega} := \sum_{\mathbf{q}} \nu_q \Delta E^{(2)}_{\omega,\mathbf{q}}. \tag{13}$$

We shall consider the contribution to the shift $\Delta E^{(1)}_{\omega}$ in Sec. IV.

The term $\Delta E^{(2)}_{\omega}$ makes a divergent contribution [7]. In the following we shall show that, if we subtract the second order self-energy of the electrons ΔE^0, the contribution to the resulting term is finite and independent from the normalization volume. This self-energy is

$$\Delta E^0=\sum_{\mathbf{q}} \nu_q \sum_{\mathbf{q}'\mathbf{k}'} \frac{|g^{\mathbf{k}'}_{\mathbf{q}',\mathbf{q}}|^2}{\epsilon_q-\epsilon_{q'}-\hbar c k'}. \tag{14}$$

Thus the *observable* contribution to the shift is

$$\Delta \widetilde{E}^{(2)}_\omega=\Delta E^{(2)}_\omega-\Delta E^0.$$

Using Eqs. (10) and (14),

$$\Delta \widetilde{E}^{(2)}_\omega=-\sum_{\mathbf{k}'}\sum_{\mathbf{q}\mathbf{q}'} \nu_q\nu_{q'} \frac{|g^{\mathbf{k}'}_{\mathbf{q}',\mathbf{q}}|^2}{\epsilon_q-\epsilon_{q'}-\hbar c k'}. \tag{15}$$

We now show that $1/V\Delta\widetilde{E}^{(2)}_\omega$ has a finite value, independent from the normalization volume. Since in low density conditions $\nu_q\nu_{q'}\cong z^2\exp[-\beta(\epsilon_{q'}+\epsilon_q)]$, we have, using the explicit expression (1) for $g^{\mathbf{k}'}_{\mathbf{q}',\mathbf{q}}$,

$$\frac{1}{V}\Delta\widetilde{E}^{(2)}_\omega=z^2\frac{e^2}{m^2c}\frac{\hbar^3}{2}\frac{1}{V^2} \times\sum_{\mathbf{k}\mathbf{q}\mathbf{q}'}\frac{(\mathbf{q}'\cdot\mathbf{u}_\mathbf{k})^2}{k}\frac{\exp(-\beta(\epsilon_{q'}+\epsilon_q))}{\hbar c k+\epsilon_{q'}-\epsilon_q}\delta_{\mathbf{k}+\mathbf{q}',\mathbf{q}}.$$

Summing over $\mathbf{q}'$ and using $(1/V^2)\Sigma_{\mathbf{qk}}\to[1/(2\pi)^6]\int d\mathbf{k}\int d\mathbf{q}$, we obtain

$$\frac{1}{V}\Delta\widetilde{E}^{(2)}_\omega=z^2\frac{e^2}{m^2c}\frac{\hbar^3}{2(2\pi)^6} \times\int d\mathbf{q}\int d\mathbf{k}\frac{(\mathbf{q}\cdot\mathbf{u}_\mathbf{k})^2}{k}\frac{\exp[-\beta(\epsilon_{|\mathbf{q}-\mathbf{k}|}+\epsilon_q)]}{\hbar c k+\epsilon_{|\mathbf{q}-\mathbf{k}|}-\epsilon_q}. \tag{16}$$

The integral over $\mathbf{k}$ can be computed in spherical coordinates (r,θ,φ) choosing $\mathbf{q}$ as polar axis; the polarization vector (normal to the polar axis) is defined by $\varphi=0$. We have ($\mu=\cos\theta$)

$$(\mathbf{q}\cdot\mathbf{u}_\mathbf{k})^2=q^2\cos^2\varphi\sin^2\theta=q^2\cos^2\varphi(1-\mu^2),$$

$$\epsilon_{|\mathbf{q}-\mathbf{k}|}\pm\epsilon_q=\frac{\hbar^2}{2m}(q^2+k^2-2qk\mu\pm q^2).$$

Calculating integral (16) over φ, then integrating over all the possible directions of $\mathbf{q}$

$$\frac{1}{V}\Delta\widetilde{E}^{(2)}_\omega=\frac{z^2e^2}{m^2c^2}\frac{\hbar^2}{2^5\pi^4}\int_0^{+\infty}dk\int_0^{+\infty}dq q^4\int_{-1}^{1}d\mu \times(1-\mu^2)\frac{\exp[-(2q^2+k^2-2qk\mu)/k_T^2]}{1+\frac{\hbar}{mc}\left(\frac{k}{2}-q\mu\right)}, \tag{17}$$

with $k_T:=\sqrt{2mk_BT/\hbar^2}$.

In order to calculate the integrals over q and k, we notice that the integrand is not exponentially zero only when the positive definite quantity $2q^2+k^2-2qk\mu$ is smaller than a few k_T^2's. This condition is satisfied only if k and q are small with respect to some k_T's. Then, if the plasma temperature is not higher than $\sim10^6$ K , we can neglect $(\hbar/mc)[(k/2)-q\mu]$ in the denominator of Eq. (17) with respect to 1 (in fact, $(\hbar/mc)[(k/2)-q\mu]=O[(\hbar/mc)k_T]=O(\sqrt{k_BT/mc^2})$, and, if $T\sim10^6$ K, $\sqrt{k_BT/mc^2}\sim10^{-2}$).

Now the integrals are Gaussian, and they can be easily evaluated . This gives

$$\frac{1}{V}\Delta\widetilde{E}^{(2)}_\omega=\frac{z^2e^2}{m^2c^2}\frac{\hbar^2}{2^9\pi^3}k_T^6.$$

Finally in the low density assumption ($z\ll1$), the density n of the plasma is

$$n=zk_T^3\frac{1}{2^3\pi^{3/2}}. \tag{18}$$

As a consequence, we have

$$\frac{1}{V}\Delta\widetilde{E}^{(2)}_\omega=\frac{n^2e^2}{m^2c^2}\frac{\hbar^2}{2^3}=\frac{n^2}{m^2c}\frac{\hbar^3}{2^3}\alpha. \tag{19}$$

If the plasma is not homogenous, this energy shift depends on the position via the electronic plasma thermodynamic parameters. As we noted in Sec. I, the dependence on the position is negligible over distances of the order of the wavelength. Thus, the observed energy shift is simply the volume average (see Sec. V below) of $\Delta\widetilde{E}^{(2)}_\omega$:

$$\hbar\,\delta\omega^{(2)}:=\frac{1}{V}\int_{\mathcal{V}}d\mathbf{x}\,\Delta\widetilde{E}^{(2)}_\omega(\mathbf{x})=\frac{\hbar^3}{2^3m^2c}\alpha\int_{\mathcal{V}}d\mathbf{x}\,n^2(\mathbf{x}). \tag{20}$$

IV. CLASSICAL CONTRIBUTIONS TO THE SHIFT

We now consider the contribution to the shift due to $\Delta E^{(1)}_\omega$. In low density conditions we have, after summing with respect to $\mathbf{q}'$ and taking the continuous limit,

$$\Delta E^{(1)}_\omega=\frac{e^2}{m^2}\frac{\hbar^3}{2\omega}z\frac{1}{(2\pi)^3}\int d\mathbf{q}\,e^{-\beta\epsilon_q}(\mathbf{q}\cdot\mathbf{u}_\mathbf{k})^2\left[\frac{1}{\hbar\omega+\epsilon_q-\epsilon_{|\mathbf{q}+\mathbf{k}|}}-\frac{1}{\hbar\omega+\epsilon_{|\mathbf{q}-\mathbf{k}|}-\epsilon_q}\right].$$

The integral can be calculated in the same approximation that leads to Eq. (19). We obtain

$$\Delta E^{(1)}_\omega=\frac{e^2}{m}\frac{\hbar}{2ck}\frac{k_BT}{mc^2}n. \tag{21}$$

Once again, if the plasma is not homogeneous,

$$\hbar\,\delta\omega^{(1)}:=\frac{1}{V}\int_V d\mathbf{x}\,\Delta E^{(1)}_\omega(\mathbf{x})=\frac{e^2}{m}\frac{\hbar}{2ck}\frac{k_BT}{mc^2}\frac{N}{V}, \tag{22}$$

where N is the total number of electrons in V.

Let us now determine the contribution coming from H''_I of Eq. (3). Using again a perturbative treatment we see that the first order term is no longer zero and its magnitude in terms of powers of the coupling parameter α is the same as that of (20), it cannot then be *a priori* overlooked.

The correction we are now studying is

$$\hbar\,\delta\omega''=\sum_{\mathbf{q}}\ \nu_q\langle\Phi_\omega\otimes\Psi_{\mathbf{q}}|\hat{H}_I''|\Phi_\omega\otimes\Psi_{\mathbf{q}}\rangle, \tag{23}$$

where $\hat{H}_I''$ is given by Eq. (5). Thus

$$\hbar\,\delta\omega''=\frac{e^2}{2mc^2}\frac{\hbar}{V}\sum_{\mathbf{q}'}\ \nu_{q'}\left\langle\Phi_\omega\otimes\Psi_{\mathbf{q}}\left|\sum_{\mathbf{q},\mathbf{q}',\mathbf{k},\mathbf{k}'}\frac{\delta_{\mathbf{q}-\mathbf{q}',\mathbf{k}-\mathbf{k}'}}{\sqrt{kk'}}\right.\right.$$
$$\left.\left.\times b_{\mathbf{q}'}^{\dagger}b_{\mathbf{q}}\otimes a_{\mathbf{k}'}^{\dagger}a_{\mathbf{k}}\right|\Phi_\omega\otimes\Psi_{\mathbf{q}}\right\rangle$$
$$=\frac{e^2}{2mc^2}\frac{\hbar}{V}\frac{1}{\omega}\sum_{\mathbf{q}'}\ \nu_{q'}=\frac{e^2}{2mc}\frac{\hbar}{k}\frac{N}{V}. \tag{24}$$

In fact $\Sigma_{\mathbf{q}'}\nu_{q'}$ is, by definition, the total number N of particles; consequently N/V is nothing else than the average density $\langle n\rangle$ of the plasma in the given volume. This gives $\delta\omega''=e^2\langle n\rangle/(2mck)$, which is precisely the classical correction to the dispersion relation due to the presence of a low density homogeneous electron plasma (whose constant density is $\langle n\rangle$), when the temperature is zero. In fact, when it is $\omega^2/c^2k^2\gg k_BT/mc^2$, we have (see [8])

$$\omega^2=\omega_P^2+c^2k^2\left(1+\frac{k_BT}{mc^2}\frac{\omega_P^2}{\omega^2}\right), \tag{25}$$

where $\omega_p=\sqrt{(ne^2/m)}$ is the plasma frequency. For $\omega_p\ll kc$ and $k_BT/mc^2\ll 1$ (nonrelativistic approximation) this gives

$$\omega=kc+\frac{1}{2}\left(1+\frac{k_BT}{mc^2}\right)\frac{\omega_p^2}{kc}=kc+\left(1+\frac{k_BT}{mc^2}\right)\frac{e^2n}{2mck}. \tag{26}$$

This is of course true when $T\sim10^6$ K, which implies $k_BT/mc^2\sim10^{-4}$.

The last term in Eq. (26) coincides with $\delta\omega''+\delta\omega^{(1)}$, where $\delta\omega^{(1)}$ gives the first order temperature correction. Like all the classical terms, it depends on N/V (V is the normalization volume of the electromagnetic field). Hence, when $V\to\infty$, such a term makes a vanishing contribution to the frequency shift.

V. DISCUSSION

The dependency of the classical terms on N/V rules them out when the measuring apparatus is outside of the plasma and the normalization volume for the electromagnetic wave is infinite. *This is not the case for the quantum contribution* (20), which is always different from zero: it will be the only contribution to the frequency shift observable in astrophysical or even cosmological conditions. An important remark on Eq. (20) is that the volume over which one integrates cannot be the volume of the whole universe; in fact it extends from the source to the receiver ($\sim$ from $-\infty$ to $+\infty$) along the line of sight, but transversely it should not be more than the distance over which the plasma may practically be thought of as infinite for quantum mechanical calculations. This transversal extension is a sort of coherence length for the plasma. It should be less than the wavelength of plasmons, the Debye length, and the screening length of the plasma. On the other hand, its square should of course be much greater than the Compton scattering cross section: our approach indeed has nothing to do with individual scattering phenomena.

Assuming for simplicity a Gaussian distribution along the line of sight, such as

$$n(x)=n_0e^{-x^2/2R^2},$$

Eq. (20) gives

$$\delta\omega^{(2)}=\frac{\hbar^2}{2^3m^2c}\alpha n_0^2\pi^{3/2}RL^2. \tag{27}$$

L is the transverse ''coherence length:'' it may in fact be used as a phenomenological parameter.

In our model relaxation phenomena in the plasma play no role. This is because we consider a steady state situation and, furthermore, our plasma is modeled as a reservoir, that can exchange with the photon any amount of energy without changing its thermodynamic state. Thus any dynamic process due to the transit of the photon is neglected. Of course, this approximation is good only if the plasma is very extended, its density is very low, and the electromagnetic field is not too strong; all of these conditions are indeed satisfied in our case.

The result we found has been obtained under some assumptions that need a careful consideration. First of all, the calculation is based on the perturbation theory. Usually this theory treats single electrons interacting with an external field or with a bath of photons. Here we are dealing with one photon interacting with a gas of electrons. The fermionic nature of electrons comes into play through the factor $\nu_{\mathbf{q}}\nu_{\mathbf{q}'}$ in Eq. (15), or, simply, when the approximation is appropriate, through the fugacity z of the plasma. When the fugacity is small enough, it is easy to see that the relevant terms of the perturbative series are weighed not only by α and its powers, but rather by products of powers of α and powers of z: this fact may change the relative importance of the different contributions.

Actually Eq. (20) is of the order αz^2. If relativistic effects had been taken into account, a second-order contribution would also have come from processes such as virtual pair creation. Such a contribution is proportional to $\alpha z(\hbar\omega/mc^2)^2$, and can be overlooked when

$$\left(\frac{\hbar\omega}{mc^2}\right)^2\ll z.$$

To sum up we conclude that the blueshift found in this paper applies to waves whose frequency satisfies to the condition

$$\omega_p<\omega\ll\frac{mc^2}{\hbar}\sqrt{z}.$$

For higher frequencies relativistic contributions must be included. To get an idea of the numbers, consider that in the physical conditions of vast portions of the solar corona z can be as low as 10^{-17}; consequently, the upper frequency is $\sim10^{11}$ Hz.

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